



Indiana State Math Contest

2023

Pre-Algebra

Exam

This test was prepared by faculty of
Indiana University
Matthias Strauch, Ph.D.

Mark your calendar:

ICTM State Awards Ceremony 2023: Friday, June 9, 2023

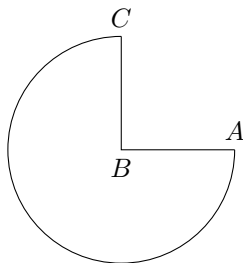
ICTM State Math Contest 2024: Saturday, April TBA, 2024

Do not open this test booklet until you have been
advised to do so by the test proctor.

Indiana Council of Teachers of Mathematics
State Mathematics Competition
Pre-Algebra 2023

this test was prepared by: Indiana University Bloomington, Department of Mathematics

1. If the length of a rectangle is increased by 10% and the width is increased by 20%, by how much will the area increase?
a. 30% b. 32% c. 40% d. 60% e. None of these
2. Rearranging the digits of 782 produces different numbers. What is the difference between the largest and the fourth largest arrangement?
a. 504 b. 144 c. 594 d. 54 e. 90
3. Jane has two siblings – a sister who is 3 years older than she and a brother who is 2 years younger than she. If the sum of the ages of the three siblings is 25, how old is Jane?
a. 4 b. 6 c. 8 d. 11 e. 13
4. Set $x = \frac{3}{0.07} - \frac{2}{0.05} + \frac{1}{0.35}$. Then, for which of the following numbers n is nx an integer?
a. 5 b. 8 c. 3 d. 7 e. 20
5. All together, 24 octagons have the same number of sides as ___?___ triangles.
a. 8 b. 32 c. 48 d. 64 e. None of
6. Claire walks her dog from her house (labeled A below) one mile to the west (to point B), then one mile to the north (to C), and then on a circular path back to her home. All together, her walk is then approximately how many miles long? (The angle ABC is 90° .)

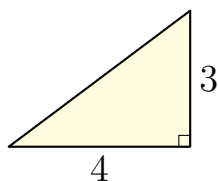


- a. 6.53 b. 6.71 c. 6.14 d. 5.92 e. 7.31

7. Sam walked from his house to a nearby grocery store at a speed of 4 mph and back to his house at a speed of 2 mph. The total time taken for making this round trip was 25 minutes, including 10 minutes of shopping at the store. How far is the grocery store from Sam's house?
- a. $\frac{1}{6}$ miles b. $\frac{1}{5}$ miles c. $\frac{1}{4}$ miles d. $\frac{1}{3}$ miles e. $\frac{1}{2}$ miles
8. Find $x > 0$ such that $(1 + \frac{1}{3})(1 + \frac{1}{5})(1 + \frac{1}{7})(1 + \frac{1}{9}) \cdot x^2 = \frac{2}{7}$.
- a. $\frac{3}{8}$ b. $\frac{7}{2}$ c. $\frac{5}{7}$ d. $\frac{2}{3}$ e. $\frac{5}{6}$
9. Hermione's potion needs 3 ingredients. Her choices are fly, beetle, snakeskin, spider, and slug. How many different combinations of 3 of these 5 ingredients are there?
- a. 6 b. 8 c. 10 d. 15 e. None of these
10. Let $z = 10$. Which is the greatest of the following numbers?
- a. \sqrt{z} b. $\frac{z^2}{1+z}$ c. $\frac{1+z}{z}$ d. $\frac{20}{z+2}$ e. $\frac{1}{z}$
11. A university charges late payment fees of 10%, compounded each month it's past due. What percent extra fees would a student end up paying if they're 4 months late?
- a. 40% b. 42% c. 43% d. 45% e. More than 45%
12. 1212 is a 4-digit number which contains the digit 1 twice and the digit 2 twice. How many whole 4-digit numbers are there with this property, namely that two of their digits are equal to 1 and the other two digits are equal to 2?
- a. 10 b. 12 c. 8 d. 6 e. None of these
13. The volume of a ball of radius r is $\frac{4}{3}\pi r^3$. A circular cross-section of that ball, which contains the center, has area πr^2 . If the area of such a cross section is 3π square inches, what is the volume of the ball?
- a. 18.51 in³ b. 20.26 in³ c. 21.77 in³ d. 23.04 in³ e. 26.78 in³
14. How many integers between 1 and 100 are divisible by 7, but not divisible by 3?
- a. 10 b. 33 c. 14 d. 29 e. 4
15. Your average score on five tests is 95. What minimum score do you need on the sixth test to get an average score of 90?
- a. 90 b. 85 c. 70 d. 65 e. 50
16. Adding the 3-digit number 1A3 to 876 gives the 4-digit number 10B9. What is the largest possible value for A such that 10B9 is divisible by 3?
- a. 8 b. 5 c. 7 d. 9 e. 6

17. Given that the point $(9, -2)$ is the midpoint of the line segment joining $(7, y)$ and $(x, 3)$, what is the value of $x - y$?
- a. 4 b. 10 c. 18 d. 22 e. None of these
18. Given the equation $2x - 14y = 4$, which of the following is a possible value of x such that y is also an integer?
- a. 10 b. 6 c. 8 d. 7 e. 9
19. A bag contains \$50 in the form of 25¢, 10¢, 5¢ coins in the ratio 4 : 5 : 10. How many 10¢ coins are there in the bag?
- a. 50 b. 75 c. 100 d. 125 e. 150
20. If x and y are positive numbers such that $x + y < x^2$, then which of the following inequalities must be true:
- a. $1 < x$ b. $y < 1$ c. $x < 1$ d. $y < x$ e. $1 < y$
21. Two angles form a linear pair. One angle is 38° more than the second angle. What is the measure of the second angle?
- a. 38° b. 71° c. 109° d. 142° e. None of these
22. Consider the sequence $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \dots, \frac{1}{81}, \frac{1}{100}$. How many terms are represented by the "..."?
- a. 3 b. 4 c. 5 d. 6 e. 7
23. A stack of 48 identical books is 8 feet high. How many such books are there in a 16 inches high stack?
- a. 32 b. 24 c. 8 d. 16 e. 4
24. Tim, Tom, and Sam compare their savings. Tim has saved \$18, Sam has saved \$8, and the ratio of Tim's savings to Tom's is the same as that of Tom's savings to Sam's savings. How much has Tom saved?
- a. 12 b. 10 c. 9 d. 11 e. None of these
25. At lunchtime each day you must choose: one sandwich from three options, one bag of chips from four options, one fruit from two options and one drink from four options. How many possible different lunch combinations are there?
- a. 36 b. 52 c. 84 d. 96 e. None of these
26. The number of natural number divisors of 105 (**including** 1 and 105) is:
- a. 7 b. 6 c. 10 d. 16 e. 8

27. A farmer has enough food to feed 20 animals in his cattle for 8 days. How long would the food last if there were 12 more cattle in his herd?
- a. 3 days b. 5 days c. 6 days d. 4 days e. 10 days
28. Let AB and CD be two 2-digit natural numbers with leading digits A and C both at least one. How many such pairs (AB, CD) are there such that $AB + CD = 99$? (Consider (AB, CD) and (CD, AB) to be different pairs, unless $AB = CD$.)
- a. 89 b. 79 c. 80 d. 81 e. 98
29. Jack bought 7 green and 8 red apples and put them all in the same bag at the grocery store. When he got home, his sister Jill reached in without looking and pulled out one apple. What is the probability that she picked a green apple?
- a. $7/15$ b. $1/2$ c. $8/15$ d. $7/8$ e. None of these
30. Let x, y, z be positive consecutive integers. Which of the following is a possible value of $x + y + z$?
- a. 34 b. 7 c. 25 d. 102 e. 79
31. There is a set of five positive integers whose mean (average) is 6, median is 6 and the only mode is 6. If the range (difference between the largest and the smallest integer) of this set is 6, what is the sum of the smallest integer and the largest integer in the set?
- a. 6 b. 9 c. 12 d. 18 e. 24
32. The sum of the positive **even** integers between 2 and 100 (including 2 and 100) is greater than the sum of the **odd** integers between 1 and 99 (including 1 and 99). By how much?
- a. 1 b. 99 c. 51 d. 100 e. none of these
33. Let P = perimeter and A = area of the triangle below, whose side lengths are as indicated. Without considering units, what is $2A + P$?



- a. 12 b. 15 c. 18 d. 24 e. 30
34. Given $\frac{2}{a} + \frac{3}{b} = \frac{1}{c}$ where $ab = 384$ and $c = 4$. What is the value of $3a + 2b$?
- a. 124 b. 96 c. $\frac{8}{5}$ d. 100 e. None of these
35. Supermarket X offers 10% discount but applies a sales tax of 4% after discount, whereas Supermarket Y offers 20% discount but applies a sales tax of 17% before discount. How much extra do you save by shopping from Supermarket Y?

- a. 6% b. 4% c. 3% d. 5% e. None of these

36. If $xy = 21$ and $\sqrt{6 + y^2} = 4$, then $x + y$ is closest to:

- a. 10 b. 9 c. 10.5 d. 11 e. 9.5

37. If a small circle's diameter is a larger circle's radius, then the area of the larger circle is how much the area of the smaller circle:

- a. 2 times b. 3 times c. 4 times d. 5 times e. None of these

38. A fundraiser runs for 100 days (including the first and last day) and it starts on a Monday. On what day of the week will it finish?

- a. Monday b. Tuesday c. Wednesday d. Thursday e. Friday

39. An after-school program has 50 students. When asked about their pets they gave the following answers: 20 have a dog, 18 have a cat, 3 have a bird and 9 have no pets at home. What is the probability that a randomly chosen student has a pet without wings?

- a. $2/5$ b. $3/50$ c. $9/25$ d. $9/50$ e. $19/25$

40. If a and b are positive integers such that $ab = 36$ and the greatest common divisor of a and b is 3, then $a + b$ is:

- a. 10 b. 9 c. 12 d. 8 e. 15