

Pre-Algebra 2022

Answers and Solutions

Answer Key:

1. A
2. B
3. C
4. B
5. D
6. A
7. A
8. B
9. B
10. C
11. D
12. A
13. D
14. B
15. D
16. C
17. D
18. B
19. D
20. E
21. A
22. A
23. B
24. D
25. D
26. C
27. B
28. A
29. B
30. A
31. C
32. B
33. D
34. A
35. D
36. D
37. C
38. B
39. D
40. D

Solutions:

1. Solution: A is the answer. We go at it by just multiplying it out, dividing, etc,

$$\frac{2^3 \times 9 \times 11}{4 \times 5 \times 20} \times \frac{4^2 \times 5^2}{8 \times 9 \times 11} = 1.$$

2. Solution: The answer is B. We immediately see

$$\begin{aligned} \frac{(0.2)^2}{2} &= \frac{0.04}{2} \\ &= 0.02 \end{aligned}$$

3. Solution: The answer is C. In four years, each of the 6 children will be 4 years older. This means the ages of the children will total $30 + (6 \times 4) = 54$

4. Solution: Answer is B, $89201.6 + 90201.8 + 91201.5 + 92201.9 + 93201.69 + 94201.71 + 95201.68 + 96201.72 + 97201.67 + 98201.73 + 99201.7 = 1036218.7$

5. Solution: Answer D, $\frac{1000 \times 75}{5} = 15000$

6. Solution: Answer is A, the least number in the set is $2x$ since

$$\begin{aligned} 2x &= 2(-2) \\ &= -4 \end{aligned}$$

$$\begin{aligned} -4x &= -4(-2) \\ &= 8 \end{aligned}$$

$$\begin{aligned} x^2 &= (-2)^2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \frac{4}{x} &= \frac{4}{-2} \\ &= -2 \end{aligned}$$

$$\frac{0}{x} = 0$$

7. Solution: Answer is A. First doing the subtraction, we get

$$\left(\frac{99}{100}\right)\left(\frac{98}{99}\right)\left(\frac{97}{98}\right)\left(\frac{96}{97}\right)\left(\frac{95}{96}\right)\left(\frac{94}{95}\right)\left(\frac{93}{94}\right)\left(\frac{92}{93}\right)\left(\frac{91}{92}\right)\left(\frac{90}{91}\right) \times \frac{1}{9} = \frac{1}{10}$$

We notice a lot of terms cancel, so the answer is $9/10$.

8. Solution: Answer B. The result is $(49/84 + 8/64)/2 = 17/48$.

9. Solution: The answer is B. We are given the three side lengths of the triangle, so we can compute the perimeter of the triangle to be $6.2 + 8.3 + 9.5 = 24$ cm. The square has the same perimeter as the triangle, so its side length is $\frac{24}{4} = 6$ cm. Finally, the area of the square is $6^2 = 36$ cm^2 .

10. Solution: The answer is C. The positive factors of 36 are: 1, 2, 3, 4, 6, 12, 18, 36

11. Solution: The answer is D since $1.1^2 = 1.21$

12. Solution: The answer is A since $(0.5 \times 4 + 0.5 \times 10) = 7$

13. The answer is D, $1234 < 1243 < 1324 < 1342 < 1423 < 1432 < \dots$

14. The answer is B, $(4k + 6k) = 1000$, $600 - 400 = 200$

15. The answer is D. These 3 prime numbers, 2, 3, 5, are not factors of 119. 7 divides 119.

16. The answer is C, the reason is 1, 2, 4, 7, 11, 16, 22, 29, ...

17. Using $z^3 + 3 = 219$, obtain $z^3 = 219 - 3 = 216$, $z = 6$, $xyz = 8z = 8 \times 6 = 48$. Answer D

18. Since $\sqrt{\frac{6010}{99} + \frac{395}{100}}$ is approximately $\sqrt{64} = 8$. The answer is B

19. The answer is D, since $209 = 11 \cdot 19$, the product is divisible by 209.

20. The answer is E, the reason is: let Ann has x dollars, then Joe has $x+15$, and Henry has $x+15+24$, together, they have $x+(x+15)+(x+15+24)=57$, solve for x , $x=1$

21. If the dimension of the solid are a , b , and c , then $ab=x$, $bc=y$, and $ca=xy$, $a^2b^2c^2 = (x)(y)(xy) = x^2y^2$. The volume is $abc=xy$. Answer A.

22. Solution: The perimeter of the rectangular garden is $2(40+20) = 120$ feet. A square with this perimeter has side length $120/4 = 30$ feet. The area of the rectangular garden is $(40)(20) = 800$ and the area of the square garden is $(30)(30) = 900$, so the area increases by $900 - 800 = 100$. The answer is A

23. Solution: The answer is B since $x = (1/5)(5/3)(3/2) = 1/2$

24. Solution: There are $150 - 30 = 120$ miles between the second and tenth exits, so the service center is at milepost $30 + (3/4)(120) = 120$. The answer is D.

25. The answer is D. Number of multiples of 2 less than 1000 = 499 ($2 \cdot 499 = 998$), Number of multiples of 3 less than 1000 = 333 ($3 \cdot 333 = 999$). Some of the integers that are divisible by both 2 and 3 are double counted. $\text{LCM}(2, 3) = 6$. Number of multiples of 6 less than 1000 = 166 ($6 \cdot 166 = 996$). Number of positive integers that are not divisible by 2 or 3 is $999 - (499 + 333 - 166) = 333$.

26. Answer is C. The probability of green is $25/60 = 5/12$

27. The answer is B. $\frac{x}{4} + \frac{y}{5} = \frac{5x+4y}{20} = \frac{18}{20}$, $x=2$ and $y=2$, thus $4x+4y=16$

28. The answer is A. $\frac{4\pi(2r)^3}{3} = 8 \frac{4\pi(r)^3}{3} = 8V$

29. The answer is B: The ratio means that for every 11 games won, 4 are lost, so the team has won $11x$ games, lost $4x$ games, and played $15x$ games for some positive integer x . The percentage of games lost is just $(4x/15x) 100 = 26.666\dots = \text{about } 27\%$

30. The answer is A. First, find the total amount of the girl's ages and add it to the total amount of the boy's ages. It equals $(15)(15) + (15)(16) = 465$. The total amount of everyone's ages can be found from the average age, $(17)(35) = 595$. Then you do $595 - 465 = 130$ to find the sum of the adult's ages. The average age of an adult is divided among the five of them, $130/5 = 26$.

31. The answer is C, since $a^2 + 1 = 3a$, then $a + \frac{1}{a} = 3$, $(a + \frac{1}{a})^2 = 3^2 = 9$, implies

$$a^2 + \frac{1}{a^2} = 9 - 2 = 7. \text{ So the answer is } 1/7$$

32. The answer is B. Solution: $\sqrt{2x+1} = \sqrt{2(2019^2 + 2020^2 - 1)} + 1 = 4039 = 2a + 1$

33. The answer is D. The reason: $x+y+xy+1=22$, then $(x+1)(y+1)=(2)(11)$, thus $(x+1)+(y+1)=11+2$, so, $x+y=11$.
34. The answer is A. Solution $b = 2d$, $c = 3d$, $b + c + 2d = 42$. Substituting: $2d + 3d + 2d = 42$, we have $7d = 42$, thus $d = 42/7 = 6$. Substituting this back: $b = 2(6) = 12$.
35. The answer is D. Solution: In bag A, there are 26 jellybeans, and 50% are yellow. That means there are $(26)(50\%)=13$ yellow jelly beans in this bag. In bag B, there are 28 jellybeans, and 25% are yellow. That means there are $(28)(25\%)=7$ yellow jelly beans in this bag. In bag C, there are 30 jellybeans, and 30% are yellow. That means there are $30\%(30)=9$ yellow jelly beans in this bag. In all three bags, there are $13+7+9=29$ yellow jelly beans in total, and $26+28+30=84$ jelly beans of all types in total. Thus, $29/84= 34.5\%$ of all jellybeans are yellow.
36. The answer is D. Solution: Recall that two angles are supplementary if and only if the sum of the two angles is 180 degrees. Let x be the smaller angle. $(x + 70) + x = 180$, $2x = 110$, $x = 55$. Therefore the smaller angle is 55 degrees.
37. Solution: The answer C. When these numbers are ordered in ascending order, 5, the median, falls right in the middle, which is the third integer from the left. Since there is a unique mode of 8, both integers to the right of 5 must be 8s. Since the mean is 5, the sum of the integers is 25, which means the 2 leftmost integers have to sum to 4. 2 and 2 does not work because that would result in two modes. However, 1 and 3 does, and so our answer is $1+5=6$.
38. The answer is B. Last week, each box was $10/8=1.25$. This week, each box is $8/10=0.8$. Percent decrease is given by $(1.25-0.8)/1.25 = 36\%$, which is closest to B.
39. The answer is D. Solution: The smallest whole number between $\sqrt{6}$ and $\sqrt{145}$ is 3. The largest whole number between $\sqrt{6}$ and $\sqrt{145}$ is 12, the number of whole numbers between $\sqrt{6}$ and $\sqrt{145}$ is 10.
40. The answer is D. Solution: Notice that the numerator of the first fraction cancels out the denominator of the second fraction, and the numerator of the second fraction cancels out the denominator of the third fraction, and so on. The only numbers left will be x in the numerator from the last fraction and 2 in the denominator from the first fraction. (The y will cancel with the numerator of the preceding number.) Thus, $x/2=10$, $x=20$. Since the

numerator is always one more than the denominator, $y=x-1=19$, thus $x+y=39$, giving an answer of D