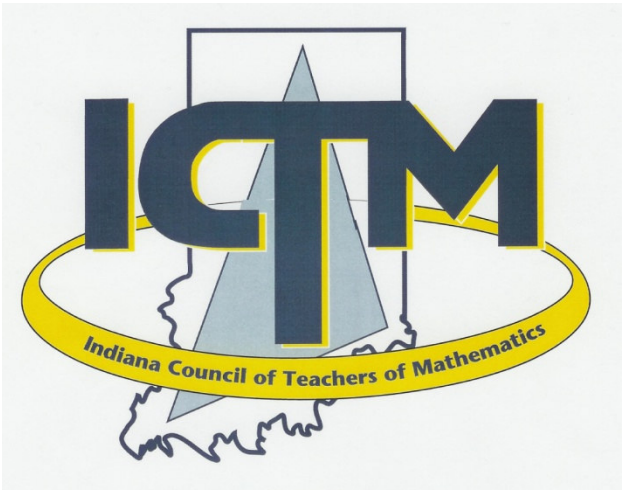

Comprehensive 2011

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **IUPUI**

ICTM Website

http://www.indianamath.org/

Do not open this test booklet until you have been advised by the test proctor.

Next year's math contest date: April 28, 2012

1. Find the inverse function. $f(x) = 2 + 3e^{4x}$

A) $f^{-1}(x) = 3\ln\left(\frac{x+2}{4}\right)$ B) $f^{-1}(x) = \frac{1}{3}\ln\left(\frac{x+2}{4}\right)$ C) $f^{-1}(x) = 3\ln\left(\frac{x-2}{4}\right)$

D) $f^{-1}(x) = \frac{1}{3}\ln\left(\frac{x-2}{4}\right)$ E) $f^{-1}(x) = \frac{1}{4}\ln\left(\frac{x-2}{3}\right)$

2. If $\begin{vmatrix} a & c \\ d & b \end{vmatrix}$ has the value $ab - cd$, then the equation $\begin{vmatrix} 2x & 1 \\ x & x \end{vmatrix} = 0$:

A) is satisfied for only 1 value of x.

B) is satisfied for 2 values of x.

C) is satisfied for no values of x.

D) is satisfied for an infinite number of values of x.

E) none of these

3. In triangle *ABC*, altitude $\overline{AH}$ and median $\overline{BM}$ intersect inside the triangle and are congruent. If the measure of $\angle ACB = 45°$, then the measure of $\angle MBC$ is:

A) $30°$ B) $35°$ C) $40°$ D) $45°$ E) $60°$

4. In an experiment, a die with six sides is repeatedly rolled and the numbers that come up on each roll are added. Once the sum *exceeds* 12, the experiment ends. For this experiment, what sum is expected to occur most frequently?

A) 17 B) 16 C) 15 D) 14 E) 13

5. The sum $\sqrt[3]{5+2\sqrt{13}}+\sqrt[3]{5-2\sqrt{13}}$ is equal to:

A) $\frac{\sqrt[3]{65}}{4}$ B) $4\sqrt{13}$ C) $\frac{3}{2}$ D) 1 E) $\frac{1+\sqrt[6]{13}}{2}$

6. A sequence $\{b_i\}$ is defined as $b_{i+1}=\frac{1}{1-b_i}$ for $i \geq 1$. If $b_8 = b_1$, compute $(b_9)^9$.

A) 0 B) -10 C) -1 D) 1 E) 10

7. The median of a trapezoid cuts the trapezoid into two regions whose areas are in the ratio $1:2$. Compute the ratio of the smaller base of the trapezoid to its longer base.

A) $1:5$ B) $1:4$ C) $1:3$ D) $1:2$ E) $1:1$

8. Select the expression that is equivalent to:

$\csc\left(\cot^{-1}\left(\frac{\sqrt{25-x^2}}{x}\right)\right)$

A) $\frac{\sqrt{25-x^2}}{x}$ B) $\frac{5}{x}$ C) $\frac{x}{\sqrt{25-x^2}}$ D) $\frac{x}{5}$ E) $\frac{5}{\sqrt{25-x^2}}$

9. Compute the number of integers from 1 through 100 inclusive that are of the form kn^2, where k and n are positive integers and $n > 1$.

A) 11 B) 25 C) 36 D) 39 E) 42

10. How many faces, edges, and vertices does a pyramid with an n-gon base have?

A) $n,\ n+1,\ n+2$ B) $n,\ 2n,\ n+1$ C) $n+1,\ 2n,\ n+1$

D) $n+1,\ n,\ 2n$ E) $2n,\ n+1,\ n+2$

11. Compute.

$$1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+\cdots+\frac{1}{16}(1+2+3+\cdots+16)$$

A) 152 B) 120 C) 76 D) 1 E) none of these

12. A bag contains 6 coins. Two coins have 2 heads (double-headed on each coin), the other four coins are fair (a head and a tail on each coin). A coin is selected at random from the bag and flipped. The coin lands with heads showing. What is the probability that the coin selected was the double-headed coin?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

13. For all ordered pairs of positive integers (x,y), we define $f(x,y)$ as follows:

$f(x,1)=x$

$f(x,y)=0$, if $y>x$

$f(x+1,y)=y\cdot[f(x,y)+f(x,y-1)]$

Compute $f(5,5)$.

A) 720 B) 120 C) 24 D) 9 E) 0

14. If $g(a) = a - 5$ and $G(a, b) = a - 2b$, then $G(-6, g(3))$ is:

A) -22 B) -8 C) -6 D) -3 E) -2

15. As the number of sides of a polygon increases from 3 to *n*, the sum of the exterior angles formed by extending each side in succession:

A) remains constant

B) increases

C) decreases

D) becomes $(n - 3)$ straight angles

E) cannot be predicted

16. A right triangle ABC has its hypotenuse AB trisected at M and N. If $CM^2 + CN^2 = k \cdot AB^2$, then what is the value of k ?

A) 2 B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $\frac{5}{9}$ E) $\frac{1}{4}$

17. If $\log_{10} m = b - \log_{10} n$, then m is equal to:

A) $\frac{b}{n}$ B) $\frac{10^b}{n}$ C) bn D) $10^b n$ E) $b - 10^n$

18. If $y = \log_a x$, and $a > 1$, which of the following statements is incorrect?

A) if $x = 1$, then $y = 0$

B) if $x = a$, then $y = 1$

C) if $x = -1$, then y is imaginary

D) if $0 < x < 1$, then y is always less than 0

E) all of the above statements are correct

19. When $x^{18}+1$ is divided by $x-1$, the remainder is:

A) -1 B) 0 C) 1 D) 2 E) x

20. Two bicyclists θ and π start at the same time to ride from point A to point B, 60 miles away. θ travels 4 miles per hour slower than π. π reaches point B and at once turns back and meets θ 12 miles from point B. The rate that θ travels is:

A) 8 mph B) 12 mph C) 16 mph D) 20 mph E) 24 mph

21. A six digit number is formed by repeating a three digit number; for example, 256,256 or 678,678. Any number of this form is always exactly divisible by:

A) 7 only B) 11 only C) 13 only D) 101 E) 1001

22. The Fibonacci sequence is defined by $F_1 = 1,\quad F_2 = 1,\quad F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Determine the smallest positive integer k so that F_k is divisible by 31.

A) 8 B) 15 C) 30 D) 60 E) none of these

23. If the radius of a circle is a rational number, then its area is given by a number which is:

A) rational B) irrational C) integral

D) rational perfect square E) none of these

24. Determine the value of B so that the line whose equation is $3x + By = 5$ is perpendicular to the line containing the points $(3,4)$ and $(-6,7)$.

A) -3 B) -2 C) -1 D) 2/3 E) 3

25. The number of significant digits in the measurement of the side of a square whose computed area is 1.1025 square inches to the nearest ten-thousandths of a square inch is:

A) 1 B) 2 C) 3 D) 4 E) 5

26. Solve for x : $e^{2x} - 11e^{x} + 30 = 0$

A) $x = \ln 5$ or $x = \ln 6$

B) $x = 5$ or $x = 6$

C) $x = e^{5}$ or $x = e^{6}$

D) $x = e^{-11}$ or $x = e^{30}$

E) $x = -\ln 11$ or $x = \ln 30$

27. Evaluate the limit:

$$\lim_{x \to \pi} \frac{x^2 - \pi^2}{x - \pi}$$

A) 0 B) 1 C) π D) 2π E) undefined

28. The equation $x - \frac{7}{x-3} = 3 - \frac{7}{x-3}$ has

A) infinitely many integral roots

B) no roots

C) one integral root

D) two equal integral roots

E) two non-equal integral roots

29. The diagonal of square I is $a+b$. The perimeter of square II which has twice the area of square I is:

A) $(a+b)^2$ B) $\sqrt{2}(a+b)^2$ C) $4(a+b)$ D) $2(a+b)$ E) $\sqrt{8}(a+b)$

30. Given $\left(10^{12}+25\right)^2-\left(10^6-25\right)^2=10^n$, find n.

A) 1 B) 2 C) 4 D) 12 E) 14

31. A circle passes through vertices $A, B,$ and D of rhombus $ABCD$ with the measure of $\angle A = 60°$. The circle intersects $\overline{AC}$ at point P. If $PC = 4$, compute AC.

A) 6 B) 8 C) 10 D) 12 E) 14

32. The product of the roots of the equation $x^2-4x+8=0$ is equal to:

A) -4 B) 0 C) 2 D) $4+4i$ E) 8