Comprehensive Test

2010

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by **Indiana University-Purdue University Indianapolis**

ICTM Website
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Next year's math contest date: April 23, 2011

1. If $f(a) = a + 3$ and $F(a,b) = a^2 - b$, then $F(-1, f(2))$ is:

(A) -6
(B) -4
(C) 3
(D) 4
(E) 5

2. A hat contains 5 coins. One coin has 2 heads (double-headed coin), the other four coins are fair (a head and a tail on each coin). A coin is selected at random from the hat and flipped. The coin lands with heads showing. What is the probability that the coin selected was the double-headed coin?

(A) 1 / 2
(B) 1 / 3
(C) 1 / 4
(D) 1 / 5
(E) 1 / 6

3. Find the domain of $f(x) = (x-3)^{1/2}(x+4)^{-1/2}$.

(A) $(-\infty,-4)\cup(3,\infty)$
(B) $(-\infty, 3]$
(C) $[3, \infty)$
(D) $(-\infty, -4)$
(E) all real numbers

4. Determine the value of A so that the line whose equation is $Ax + y = 2$ is perpendicular to the line containing the points $(1,-3)$ and $(-2,4)$.

(A) 11 / 7
(B) $-7/3$
(C) $-1/2$
(D) $-3/7$
(E) 3

5. The sum of the roots of the equation $x^2 - 4x + 8 = 0$ is equal to:

(A) 4
(B) 2
(C) -2
(D) $2+2i$
(E) $4+4i$

6. An equilateral triangle is inscribed in a circle of area 400π. Find the area of the triangle.

(A) $400\sqrt{2}$
(B) $800\left(\frac{\sqrt{3}}{3}\right)$
(C) $300\sqrt{3}$
(D) 600
(E) $200\sqrt{6}$

7. Solve for x: $e^{2x} - 11e^{x} + 30 = 0$

(A) $x = \ln 5$ or $x = \ln 6$
(B) $x = 5$ or $x = 6$
(C) $x = e^5$ or $x = e^6$
(D) $x = e^{-11}$ or $x = e^{30}$
(E) $x = -\ln 11$ or $x = \ln 30$

8. Select the expression that is equivalent to: $\cos\left(\arctan\frac{x}{3}\right)$

(A) $\frac{\sqrt{x^2-9}}{x}$
(B) $\frac{x}{\sqrt{x^2+9}}$
(C) $\sqrt{x^2-9}$
(D) $\frac{3}{\sqrt{x^2-9}}$
(E) $\frac{3}{\sqrt{x^2+9}}$

9. The sum $\sqrt[3]{5+2\sqrt{13}}+\sqrt[3]{5-2\sqrt{13}}$ equals

(A) 0
(B) 1
(C) $3/2$
(D) $\dfrac{\sqrt[3]{65}}{4}$
(E) $\dfrac{1+\sqrt[6]{13}}{2}$

10. Consider the two points on the x-axis: $(-3,0)$ and $(3,0)$. The set of all points, such that the sum of the distances to each of the given points equals 10 may be represented by which of the following equations.

(A) $x^2+y^2=25$
(B) $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$
(C) $\dfrac{x^2}{25}-\dfrac{y^2}{16}=1$
(D) $\dfrac{x^2}{16}+\dfrac{y^2}{25}=1$
(E) $\dfrac{x^2}{16}-\dfrac{y^2}{9}=1$

11. For each real number x, let $f(x)$ be the minimum of the numbers $4x+1$, $x+2$, and $-2x+4$. Find the maximum value of $f(x)$.

(A) $1/2$
(B) $1/3$
(C) $2/3$
(D) $5/2$
(E) $8/3$

12. In a geometric sequence of real numbers, the sum of the first two terms is 7, and the sum of the first six terms is 91. Find the sum of the first four terms.

(A) 28
(B) 32
(C) 35
(D) 49
(E) 84

13. How many of the first one hundred positive integers are divisible by all of the numbers 2, 3, 4, 5?

(A) 4
(B) 3
(C) 2
(D) 1
(E) 0

14. The perimeter of a semicircular region, measured in meters, is numerically equal to its area, measured in square meters. Find the radius of the semicircle, measured in meters.

(A) π
(B) $2/\pi$
(C) 1
(D) $1/2$
(E) $\frac{4}{\pi}+2$

15. If the operation $x*y$ is defined by $x*y=(x+1)(y+1)-1$, then which one of the following is FALSE?

(A) $x*y=y*x$ for all real numbers
(B) $x*(y+z)=(x*y)+(x*z)$ for all real numbers
(C) $(x-1)*(x+1)=(x*x)-1$ for all real numbers
(D) $x*0=x$ for all real numbers
(E) $x*(y*z)=(x*y)*z$ for all real numbers

16. Solve the inequality. $\dfrac{x^2+5x+3}{x-1} \ge 1$

(A) $(1,\infty)$
(B) $[1,\infty)$
(C) $\{-2\}\cup(1,\infty)$
(D) $(-\infty,-1)$
(E) $\left(-\infty,\dfrac{-5-\sqrt{13}}{2}\right]\cup\left[\dfrac{-5-\sqrt{13}}{2},\infty\right)$

17. Find the inverse function. $f(x)=5\ln(x+2)$

(A) $f^{-1}(x)=\dfrac{1}{5\ln(x+2)}$

(B) $f^{-1}(x)=5e^{x-2}$

(C) $f^{-1}(x)=\dfrac{1}{5}e^{x+2}$

(D) $f^{-1}(x)=e^{x/5}-2$

(E) An inverse function does not exist for this function.

18. Solve for t: $s=-16t^2+v_0t$

(A) $t=\dfrac{v_0\pm\sqrt{v_0^2-64s}}{32}$

(B) $t=\dfrac{-v_0\pm\sqrt{v_0^2+64s}}{2}$

(C) $t=\dfrac{1\pm\sqrt{1+64s}}{32}$

(D) not enough information

(E) $t=\dfrac{-16\pm\sqrt{v_0-4s}}{2s}$

19. A regular tetrahedron has six sides all of length 1. It sits on level ground on one of its faces. What is the height of the tetrahedron?

(A) $\frac{\sqrt{3}}{2}$
(B) $\frac{\sqrt{15}}{6}$
(C) $\frac{2}{3}$
(D) $\frac{\sqrt{6}}{3}$
(E) $\frac{\sqrt{2}}{2}$

20. If θ is a constant, such that $0 < \theta < \pi$, and $x + \frac{1}{x} = 2\cos\theta$, then for each positive integer n, $x^n + \frac{1}{x^n}$ equals

(A) $2\cos\theta$
(B) $2\cos n\theta$
(C) $2\cos^n\theta$
(D) $2^n\cos\theta$
(E) $2^n\cos^n\theta$

21. The square of an integer is called a perfect square. If x is a perfect square, what is the next larger perfect square?

(A) $x+1$
(B) x^2+1
(C) x^2+2x+1
(D) x^2+x
(E) $x+2\sqrt{x}+1$

22. If $y = \log_a x$, and $a > 1$, which of the following statements is incorrect?

(A) if $x = 1$, then $y = 0$
(B) if $x = a$, then $y = 1$
(C) if $x = -1$, then y is imaginary
(D) if $0 < x < 1$, then y is always less than 0
(E) only some of the above statements are correct

23. Which of the following is ***NOT*** a solution to the given equation? $x^3 = 1$

(A) $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

(B) $\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

(C) $\frac{-1+\sqrt{-3}}{2}$

(D) $\frac{-1+\sqrt{3i}}{2}$

(E) $\frac{-1-i\sqrt{3}}{2}$

24. Which of the following is equivalent to the given expression. $\frac{1-\cos\theta}{\sin\theta}$

(A) $\frac{1+\sin\theta}{\cos\theta}$

(B) $\frac{\cos\theta}{1-\sin\theta}$

(C) $\frac{\sin\theta}{1+\cos\theta}$

(D) $\frac{1-\sin\theta}{\cos\theta}$

(E) $\frac{\sin\theta}{1-\cos\theta}$

25. Consider the equation $x^2 + bx + c = 0$. It's discriminant is given by $b^2 - 4ac$. Suppose you know that the roots of the equation $x^2 + bx + c = 0$ are r and s. An alternative expression for the discriminant would be:

(A) $(r-s)^2$

(B) $r+s$

(C) $\sqrt{r^2+s^2}$

(D) $|r-s|$

(E) rs