Algebra II / Integrated Math III
2010

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana State University**

ICTM Website
http://www.indianamath.org/

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Next year's math contest date: April 23, 2011

1) Solve for x: $\dfrac{x+3}{x-1} > 0$

   A) $x < -3$ or $x > 1$

   B) $-3 < x < 1$

   C) $x \le -3$ or $x > 1$

   D) $-3 \le x < 1$

   E) none of these

2) $(x - y + z)^2$ is equal to:

   A) $x^2 + y^2 + z^2$

   B) $x^2 + y^2 + z^2 - 2xy - 2yz + 2xz$

   C) $x^2 + y^2 + z^2 - xyz$

   D) $x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$

   E) $x^2 + y^2 + z^2 - xy - yz + xz$

3) Write the following without a radical in the denominator: $\dfrac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

   A) $\dfrac{5 - \sqrt{15}}{8}$ B) $\dfrac{\sqrt{3}}{3}$ C) $\dfrac{5 + \sqrt{3}}{3}$ D) $\dfrac{5 - \sqrt{15}}{2}$ E) none of these

4) The eccentricity of the ellipse with equation $\dfrac{y^2}{9} + \dfrac{x^2}{4} = 1$ is:

   A) $\dfrac{\sqrt{65}}{9}$ B) $\dfrac{4}{9}$ C) $\dfrac{\sqrt{5}}{3}$ D) $\dfrac{2}{3}$ E) 0

5) The units digit of 7^{2010} is:

   A) 1   B) 3   C) 5   D) 7   E) 9

6) If you write the repeating decimal $.0\overline{216}$ as a fully reduced common fraction, the difference between the denominator and the numerator is:

A) 9774   B) 1213   C) 783   D) 181   E) 29

7)   The radius of the circle with equation $x^2 + y^2 + 14x - 6y = 6$ is

A) 6   B) $\sqrt{6}$ C) 8   D) $\sqrt{8}$ E) $\sqrt{46}$

8)   Solve the equation for r in terms of s: $s = \dfrac{3r - 2}{r + 1}$

A) $\dfrac{s + 2}{3 - s}$ B) $\dfrac{3s - 2}{s + 1}$ C) $\dfrac{s + 1}{3s - 2}$ D) $\dfrac{s - 1}{3s + 2}$ E) none of these

9)   Solve the equation for x in terms of y : $y = 3e^{2x-1} + 5$, given that $y > 5$.

A) $\dfrac{1}{3}e^{2y+1} - 5$

B) $\ln\left(\dfrac{y - 5}{3}\right) + 1$

C) $\dfrac{1}{2}\left(\ln\left(\dfrac{y - 5}{3}\right) + 1\right)$

D) $3\ln(2y - 1) + 5$

E)   none of these

10)   Define an operation, $*$, for positive real numbers as $a * b = ab + 1$. Which of the following statements are true?

I. $*$ is commutative   II. $*$ is associative   III. $*$ has an identity element

A) I only   B) I and II only   C) II and III only   D) II only   E) I, II, and III

11)   On Rossana's visit to the aquarium, she saw an exhibit containing only octopi, crabs, and starfish. She counted 84 legs and 11 heads. If octopi have one head and 8 legs, crabs have one head and 6 legs, and starfish have zero heads and 5 legs, what is the product of the number of animals she saw in the exhibit if she saw at least one of each type of animal?

A) 13   B) 30   C) 48   D) 54   E) 56

12) Simplify the following rational expression:

A) $\dfrac{-1}{x+y}$　　　B) $\dfrac{1}{x}-\dfrac{1}{y}$　　　C) $\dfrac{x-y}{x^2-xy+y^2}$　　　D) $\dfrac{x-y}{x^2+xy+y^2}$　　　E) none of these

13) Consider the equation: $x^4+1=0$. The sum of all the roots of this equation is:

A) 0　　　B) 1　　　C) 4　　　D) i　　　E) $4i$

14) $\log_5 625^{10} =$

A) 12.5　　　B) 40　　　C) 50　　　D) 125　　　E) none of these

15) $3^2 - 3^{-2} =$

A) 81　　　B) $\dfrac{82}{9}$　　　C) $\dfrac{80}{9}$　　　D) 1　　　E) 0

16) $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \dfrac{1}{16} - \dfrac{1}{32} + \cdots + \dfrac{1}{2^{1,000}} =$

A) $2 - \dfrac{1}{2^{1,000}}$　　　B) $2 + \dfrac{1}{2^{1,000}}$　　　C) $\dfrac{1}{3}\left(2 - \dfrac{1}{2^{1,000}}\right)$　　　D) $\dfrac{1}{3}\left(2 + \dfrac{1}{2^{1,000}}\right)$　　　E) none of these

17) The equation of a parabola with focus $(1,0)$ and directrix $3x+4y=0$ is

A) $y = x^2 + 1$

B) $(x-1)^2 = 4y$

C) $3x^2 + 4y - 1 = 0$

D) $4x^2 + 24xy + 11y^2 + 10x - 5 = 0$

E)   none of these

18)   If the graphs of the lines with equations $3x - 2y + 4 = 0$ and $7x + ay - 1 = 0$ are perpendicular, then a is equal to:

A) $\dfrac{21}{2}$ B) $-\dfrac{2}{21}$ C) $-\dfrac{3}{7}$ D) $\dfrac{7}{3}$ E) none of these

19)   The sum of the solutions of the equation $|x - 5| = |4x + 9|$ is:

A) $\dfrac{11}{4}$ B) 0 C) $-\dfrac{82}{15}$ D) -8 E) none of these

20)   When x is added to the numerator and the denominator of the fraction $\dfrac{a}{b}$, where $b \neq 0$ and $a \neq b$, the value of the fraction is changed to $\dfrac{c}{d}$. x is equal to:

A) $\dfrac{ad - bc}{c - d}$ B) $\dfrac{ad - bc}{d - c}$ C) $\dfrac{ad - c}{b}$ D) $\dfrac{ad + bc}{c - d}$ E) none of these

21)   If the system $\begin{cases} x^2 + y^2 = 4 \\ y = kx + 3 \end{cases}$ has exactly one solution, then k is equal to:

A) 0 or -3   B) $\pm\dfrac{\sqrt{5}}{2}$ C) ± 12 D) 0   E) none of these

22) $f(x) = \dfrac{x^{17} + x^9}{x^5 - 1}$ and $i = \sqrt{-1}$. The value of $f(i)$ is:

A) $i - 1$ B) $1 + i$ C) $1 - i$ D) $2i$ E) none of these

23)   The sum of the solutions to the equation $12 - 8 \cdot 5^x + 25^x = 0$ is:

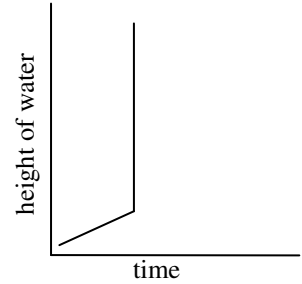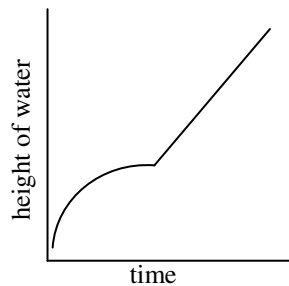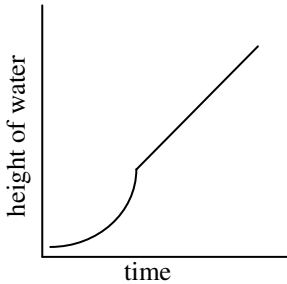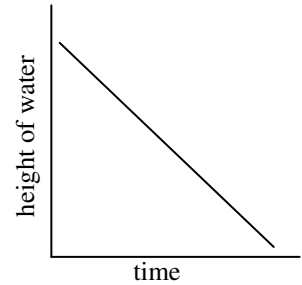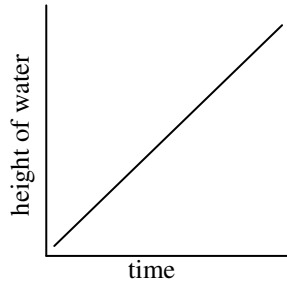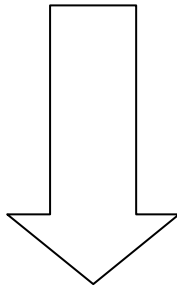A) $\log_5 12$ B) $\log_{25} 6$ C) 6   D) 17   E) none of these

24) To make a fruit punch, you mixed $\frac{1}{4}$ cup grape juice concentrate with $1\frac{1}{2}$ cups soda water. If you want to make the same punch with 2 cups of soda water, then how many cups of grape juice concentrate should you use?

A) $\frac{1}{3}$ cup   B) $\frac{3}{8}$ cup   C) $\frac{1}{2}$ cup   D) $\frac{2}{3}$ cup   E) none of these

25) The domain of the function $f(x) = \sqrt{5 - 2x}$ is:

A) $x \geq \frac{5}{2}$ B) $x \leq \frac{5}{2}$ C) $x \leq 0$ D) $x \geq 0$ E) none of these

26) The number of cubic centimeters in the volume of a sphere is the same as the number of square millimeters in its surface area. The radius of the sphere in millimeters is:

A) 300   B) 30   C) 3   D) 0.03   E) none of these

27) If $f(x) = x^2 - 3$ and $g(x) = 5 - x$, then $f(g^{-1}(x))$ is equal to:

A) $x^2 + 10x + 22$ B) $x^2 - 10x + 22$ C) $5 - \sqrt{x+3}$ D) $5 + \sqrt{x-3}$ E) none of these

28) The graph of $\dfrac{y^2}{36} + \dfrac{x^2}{4} = 0$ is

A) an ellipse

B) a point

C) a hyperbola

D) two intersecting lines

E) none of these

29) x, $2x$, $x - 8$ are the first three terms of a geometric sequence. The sum of the first four terms is:

A) $\dfrac{712}{3}$ B) 24   C) -24   D) $\dfrac{440}{3}$ E) none of these

30) The polynomial $p(x) = x^3 - 5x^2 + x - 5$ has

  A) 3 integer zeros

  B) 1 integer and 2 non-integer rational zeros

  C) 1 integer and 2 complex (non-real) zeros

  D) 3 irrational zeros

  E) none of these

31) Two positive integers have a sum of 65 and their squares have a difference of 585. What is their positive difference?

  A) 7   B) 8   C) 9   D) 10   E) none of these

32) Izak can mow a soccer field with his lawn mower in 6 hours. Mikaila can mow the same field with her lawn mower in 3 hours. How long will it take the two of them working together to mow the field?

  A) 1 hour 30 minutes

  B) 2 hours

  C) 2 hours 15 minutes

  D) 4 hours 30 minutes

  E) none of these

33) A box contains 5 red marbles and 7 blue marbles. If you draw two marbles at random from the box, what is the probability that you have drawn two marbles of the same color?

  A) $\dfrac{31}{66}$ B) $\dfrac{5}{33}$ C) $\dfrac{7}{22}$ D) $\dfrac{5}{11}$ E) none of these

34) In how many different ways can all the letters in INDIANA be arranged in a line? Assume that duplicate letters are indistinguishable.

A) 5040   B) 2520   C) 1260   D) 630   E) none of these

35) The following bottle will be filled with water at a constant rate. Which graph is the best model for the height of the water in the vase with respect to time?

36) On Monday, it took Sophie 20 minutes to walk the 10 blocks to school. On Tuesday, she was tired and it took her 30 minutes to walk the 10 blocks to school. On Wednesday, she was late, and it took her only 10 minutes to walk the 10 blocks to school. What was her average speed, in blocks per hour in her travels to school over the three days?

A) 25   B) 30   C) 36   D) 60   E) none of these